Accelerometer Noise

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The various noise sources of piezoelectric transducers connected to preamplifiers are discussed. Low level, low frequency measurements are limited by the electromechanical efficiency of the piezoelectric material used in the accelerometer and the characteristics of the preamplifier.

Noise, in general, is any undesired signal. Here, noise is limited to that generated by the electrical components of an accelerometer preamplifier which degrades accelerometer performance. Figure 1 shows the input stage of a typical preamplifier for a piezoelectric accelerometer operating as a voltage or charge amplifier. C is the capacitance of the piezoelectric element with a loss factor η which is equivalent to the inverse of the quality factor, R_1 is the biasing resistor and the active device is either a Junction Field Effect Transistor (JFET) or a Metal Oxide Semiconductor Field Effect Transistor (MOSFET). The loss factor is the sum of the loss factor of the piezoelectric material and the mechanical assembly. It may range from 0.05 to 0.001. A low loss factor may give rise to accelerometer ringing which may overload the preamplifier at the resonance frequency of the accelerometer.

The power spectral density of the preamplifier circuit above the low frequency cutoff, such that $\omega R_1 C > 1$ is:

$$E_n^2 = \frac{4kT}{\omega^2 C^2 R_1} + E_t^2 + \frac{I_t^2}{\omega^2 C^2} + \frac{4kT\eta}{\omega C} \quad \text{since } \eta \ll 1 \tag{1}$$

where k is Boltzmann's constant, 1.38×10^{-23} watt-second/kelvin, T is the absolute temperature in kelvin, E_t is the transistor noise voltage per root Hz, I_t is the transistor noise current per root Hz, and ω equals the angular frequency $2\pi f$ in radians/second.

Let R_2 be the value of a resistor having a Johnson (thermal) noise equal to the voltage noise generated by the current noise times its resistance, such that

$$4kTR_2 = I_t^2 R_2^2$$

$$R_2 = 4kT/I_t^2$$

$$I_t^2 = 2qI_{GSS}$$

where q equals the electronic charge of 1.6×10^{-19} coulomb and I_{GSS} is the gate leakage current of the transistor. Thus $R_2 = 4kT/2qI_{GSS} = 0.05/I_{GSS}$ at room temperature.

It should be noted that I_{GSS} doubles for each temperature increase of about 10° C or about thousandfold between room temperature and 120° C and in a JFET is dependent on the drain gate voltage due to impact ionization. Typically I_{GSS} for a JFET at room temperature is less than 300 fA resulting in a value for R_2 greater than 167 GΩ. R_2 for low signal MOSFETs at room temperature is about 1 TΩ.

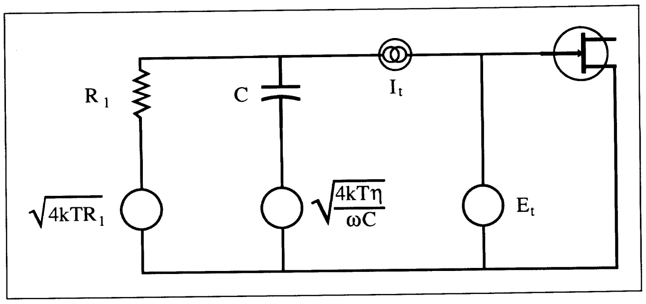

Figure 1. Input stage of a typical amplifier showing one current and three voltage noise sources.

Equation (1) can then be rewritten:

$$E_n^2 = \frac{4kT}{\omega^2 C^2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + E_t^2 + \frac{4kT\eta}{\omega C} \tag{2}$$

E_t^2 is inversely proportional to frequency below about 1 kHz called the 1/f noise. The first term is inversely proportional to the square of the frequency and thus controls the low frequency noise. Since the biasing resistor R_1 must have a smaller value than R_2 the noise at low frequencies is predominantly dependent on the passive components R_1 and C. If R_1 is too large, the low frequency response will be temperature dependent.

Let V be the voltage and Q be the charge sensitivity of the accelerometer. $Q = VC$ and let A_n^2 be the power spectral density in terms of acceleration. Then Equation (2) yields:

$$A_n^2 = \frac{4kT}{\omega^2 Q^2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + E_t^2 + \frac{E_t^2 + 4kT\eta(\omega C)^{-1}}{V^2} \tag{3}$$

It can be seen that at low frequencies the acceleration noise is inversely proportional to the charge sensitivity. It may seem that a MOSFET having a higher input impedance would be quieter at low frequencies. However the MOSFET must be protected from destruction by voltage overloads using devices having greater leakage than the transistor. In addition the MOSFET has at least ten times higher noise voltage than a low noise N channel JFET and should never be used for very quiet preamplifiers. P channel JFETs are slightly noisier than N channel JFETs.

At higher frequencies, the second term of Equation (3) controls the acceleration noise. A higher voltage sensitivity is desirable, provided that the values of all other characteristics are the same. Piezoelectric materials with a high voltage sensitivity have a much lower dielectric constant not sufficiently offset by reduction in the loss factor, so that η/C the second part of the numerator of the last term of Equation (3) is higher

Table 1. Characteristics of piezoelectric materials.

Material	Electro-Mechanical Efficiency (in Percent)	Charge Per Unit Force in pC/N (Compression)	Electric Field Per Unit Pressure in mV M/N (Compression)	Relative Dielectric Constant	Curie Temperature (in °C)
Rochelle Salt	84	500	160	350	(55)
Lead Zirconate Titanate PZT	50	350	25	1700	350
Barium Titanate	23	150	15	1200	120
Lead Metaniobate	12	75	35	240	550
Lithium Niobate	10	21	28	84	1150
Ammonium Dihydrogen Phosphate ADP	7.8	24	177	15	LOW
Bismuth Titanate	2.0	16	10	165	660
Polyvinylidene Flouride PVDF	1.8	22	190	13	(160)
Tourmaline	0.8	1.9	30	7	840
Quartz	0.7	2.2	50	4.5	573

(   ) = Melting Point

Corrections to be Applied to the Optical Calibration of Accelerometers

An accelerometer may be calibrated using the fringe disappearance method, where the sensor is placed in the center of a vibrating surface with small mirrors surrounding the base of the accelerometer. The mirrors serve as the reflecting surfaces for the laser. This calibration method cannot sense the actual movement of the accelerometer mounting point unless the table is infinitely rigid and therefore a relative motion error exists between the mirror and the mounting area – the error increasing with the square of the frequency.[1]

If the radius of the accelerometer is small compared to the radius of the vibrating table, then the mass of the accelerometer is distributed over a part of the boundary of a semi-infinite solid. The average deflection due to a uniform distribution is:

$$\frac{16}{3\pi^2}\left(\frac{P(1-\nu^2)}{aE}\right) \tag{1}$$

where P is the force, a is the radius of the load, and E and ν are the Young's modulus and Poisson's ratio of the semi-infinite solid.[2]

If the center of the mirrors is placed a distance from the center equal to 1.2 times the radius of the accelerometer base, the magnitude of the deflection is 0.25 times the bracketed part of Equation (1). Since the force P equals the mass of the accelerometer times its acceleration, the ratio of the average displacement of the accelerometer to the displacement of the mirrors is for small corrections:

$$\left(\frac{16}{3\pi^2} - 0.25\right)\left(\frac{M(1-\nu^2)}{aE}\right)\omega^2 \tag{2}$$

where M is the mass of the accelerometer and ω is the angular frequency $= 2\pi f$.

Using an accelerometer weighing 20 gms, with radius of 6.35 mm at its base attached to an alumina table of modulus 3.4×10^{11} N/m^2, a Poisson's ratio of 0.3, and having the center of the mirror 1.6 mm from the periphery of the accelerometer base, the correction to be made to the optically measured values is –0.96% at 10 kHz and –3.8% at 20 kHz. In the above derivation it is assumed that the elastic modulus of the accelerometer base material is much lower than the modulus of the table material. The correction increases by a factor of 1.7 for an accelerometer base material of infinite modulus which would be an upper bound for the correction.

These findings may be of interest only to purists since there are many more significant practical limitations to accurate measurements at high frequencies.[3]

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[1] Workman, David R., "Methods for Calibration of Vibration Measurement Reference Standards," Martin Marietta, Denver, CO, 1987.

[2] Timoshenko, S., *Theory of Elasticity, First Edition, Ninth Impression*, McGraw-Hill, Chapter 11, pg. 333, 1934.

[3] Barrett, Richard M., Schloss, Fred, "High Frequency Vibration Measurement Limitations," Proceedings Institute of Environmental Sciences, April, 1990.

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for these materials than for PZT materials. For example, the acceleration noise for a quartz accelerometer in the midfrequency region of about 5 kHz using a very low noise JFET is actually greater than a sensor using PZT despite the fact that quartz has twice the voltage sensitivity of PZT. Transducers having a low capacitance as a result of low dielectric constants have their voltage sensitivities considerably reduced due to shunting by the input capacitance including the Miller effect of the input transistor plus any stray capacitance. Further, operation to very low frequencies with low capacitance sources necessitates the use of very high input impedance MOSFETs which are noisy.

According to Equation (3), low frequency noise can be lowered by increasing the value of the biasing resistor R_1, provided $R_2 > R_1$. However, measurements have shown that the low frequency noise increases as R_1 is increased beyond a certain value. It has been found that noise in excess of thermal noise, called excess noise, generated by the leakage current flowing through the biasing resistor causes an additional noise source. It is caused by microarcs and its power is inversely proportional to frequency. It increases with increase in resistance values, decreases with the power rating of the resistor and depends on the manufacturer, material and process. Excess noise is largest for carbon resistors and smallest for wirewound resistors. The power spectral density of the excess noise is:

$$\frac{NI^2 \times 10^{-12}}{2.303f} \quad I_{GSS}^2\, R_1^2$$

where NI is the noise index expressed in μV/V per frequency decade, f being the frequency.

Thus the power spectral density of the excess noise at 1 Hz = $4.34 \times 10^{-13}\,(NI\,I_{GSS}\,R_1)^2$. The power spectral density of the thermal noise of this resistor is $4kTR_1$.

NI for a 100 GΩ, 60 mW chip resistor was measured to be 2500 μV/V resulting in a power spectral density of 2.4×10^{-9}, whereas the value due to thermal noise is 1.6×10^{-9}. However, the NI of a tubular 250 mW resistor was ten times lower. Since I_{GSS} increases exponentially with temperature, the excess noise may dominate the noise at low frequencies.

Table 1 lists, in descending order of electromechanical efficiency, materials which have been used for accelerometers. The values for the second column, charge per unit force in compression, is directly proportional to the charge sensitivity Q of Equation (3) and the third column lists the electric field per unit pressure which is proportional to the voltage sensitivity V. The Curie temperature is the temperature at which the materials lose their piezoelectric properties. Rochelle salt has the highest efficiency but unfortunately it melts at 55° C. Lead zirconate titanate (PZT), with its many formulations to achieve different desirable characteristics for a great number of applications, has an efficiency of about 50% and as a result of its high charge sensitivity, is the preferred material for low frequency, low noise accelerometers.

Practical Considerations

Low noise at low frequencies is important for industrial use, since the acceleration levels are very low, for example, 100 μg or 1 mm/sec^2 at 1 Hz. Noise at high frequencies is unimportant except for leak detection and other unusual applications. To detect low frequency levels, the electrical noise level must be lower, about 10 μg for a signal-to-noise level of 10 (20 dB). From Equation (3) using a value of 1 GΩ for the two resistors in parallel, the required charge sensitivity of the accelerometer is 63 pC/g, which is a value about that of an accelerometer with a resonance frequency of 30 kHz using PZT. Lowering the resonance frequency to 1 kHz, charge sensitivities of 15,000 pC/g have been achieved.

Present day industrial accelerometers use piezoceramics, lithium niobate, quartz or PVDF for the transduction, though other materials such as ADP, rochelle salt and others have been used in the past. Since the lead zirconate titanate (PZT) ceramics have an efficiency of converting mechanical to electrical energy 75 times greater than quartz, the attainable charge sensitivity for the PZT transducers is about 100 times greater for transducers of identical designs – that is identical resonance frequency, inertial masses and number of piezoelectric elements. The voltage sensitivities of lead metaniobate, polyvinylidene fluoride, tourmaline and quartz are greater than that of the lead zirconate titanates. However, as previously discussed, higher voltage sensitivity, when considering all aspects, does not necessarily provide lower noise at high frequencies. Due to the advantages of piezoceramics, many manufacturers of broad frequency low noise accelerometers, as well as sonar transducers, medical sensors, beepers, igniters, etc., use PZT as the piezoelectric material. ■